Implementation of an Algorithm for the Estimation of the Sea **Clutter Distribution and Parameters**

Implementación de un algoritmo para la distribución y parámetros del eco marino

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Abstract

Sea clutter is the main interfering signal in radar systems. In order to develop appropriate strategies for clutter suppression, an algorithm able to identify the distribution of radar readings becomes necessary. By using several popular methods found in the related literature, the authors design an algorithm able to identify the clutter distribution and its corresponding parameters. The proposed solution, which included the widely used maximum likelihood method and the Kolmogorov-Smirnov statistic test, was implemented in a software application with an intuitive graphical interface. As a result, a viable instance of the algorithm became available for educational and research purposes, particularly as a comparative base for estimating the effect of the size increase in the sample set when estimating the probability distribution.

Keywords: Sea clutter, estimation of probability distributions, Maximum Likelihood, Kolmogorov-Smi.

Resumen

El clutter marino es la principal señal interferente en sistemas de radar. Para desarrollar estrategias apropiadas de supresión de clutter es necesario contar con un algoritmo capaz de identificar la distribución de las lecturas de radar. Usando varios métodos populares encontrados en la literatura relacionada, los autores confeccionaron un algoritmo capaz de identificar el modelo y parámetros del clutter. La solución propuesta, que incluyó los ampliamente usados método de Máxima Verosimilitud y prueba estadística Kolmogorov-Smirnov, fue implementada



en una aplicación con interfaz gráfica intuitiva. Así, se habilitó el empleo del algoritmo con fines pedagógicos e investigativos, particularmente como base comparativa del efecto del incremento del tamaño de la muestra sobre la estimación de la distribución de probabilidad.

Palabras clave:: Clutter marino, estimación de distribuciones de probabilidad, máxima verosimilitud, Kolmogorov-Smirnov.

1. INTRODUCTION

The purpose of active radars is to detect targets within the observation area and to estimate basic parameters such as the targets' position and velocity (1). In real operating environments an interfering signal, known as clutter, appears mixed with targets and noise information, often exceeding the magnitude of the latter (2). The clutter is the result of the rebound of the radar emission on a background object which surrounds the target. Two of the most common backgrounds are ground and sea clutter (3). The sea clutter generally has a higher level of interference when compare with ground clutter, so radars operating at sea environments have serious limitations on performance imposed by unwanted echoes (4).

Sea clutter characteristics may fluctuate in a wide range. The radar designer needs to understand the fluctuations in order to be able to develop appropriate signal processing strategies and to predict performance under different conditions. An important contribution towards this subject is the development of accurate models for clutter returns. The parameters for which models are developed are (5): RCS (Radar Cross Section), electromagnetic spectrum, the discreet polarization matrix and polarization dispersion matrix, and amplitude distribution of clutter.

Specifically, the amplitude distribution of sea clutter is one of the most studied characteristics. A lot of research has been devoted to processing sea clutter samples and identifying a probabilistic model that accurately represents them (6-10). As a result, it has been determined that the roughness of sea surface defines the properties of the clutter distribution.

Additionally, it is often assumed that the roughness can be modeled using two types of waves: gravitational and capillary (11). The wavelength is the main difference between them; in the order of meters for gravitational waves and of a few centimeters for capillary waves. This compound mechanism has been used to model sea clutter empirical data by several authors (12-15). Particularly, the K distribution (16,17) is widely used in such type of modeling.

Nevertheless, there are other distributions that do not rely on the existence of gravitational and capillary waves and have been widely used for clutter modeling. This is the case of the known Log-Normal (18, 19) Weibull (20, 21) and Rayleigh (6, 13) models, whose fit with modern radar data is justified by means of the similarity between the model and the measurement and has no direct relation to the physical representation of the phenomenon.

Definitely, Rayleigh, Log-Normal, Weibull and K models are the most popular distributions even though there are others such as Log-Weibull (18), Pareto, Compound Gaussian (14) and KK (23) which have also been applied with success. While the model selection is essential in clutter representation, it is equally important to appropriately choose the model parameters. The Log-normal, Weibull and K are bi-parametric distributions and the Rayleigh model has a single parameter.

1.1. Motivation and objetives

The radar research team from the Instituto Superior Politécnico José Antonio Echeverría (ISPJAE) focuses on the creation of innovative solutions for recognizing sea

clutter parameters using artificial neural networks (24, 25, 26). While offered solutions achieve high accuracy and low computational cost, they require a comparative base to assess the exact achieved gain. Therefore, the authors set as objective for the current project the implementation of an algorithm for estimating the clutter distribution and parameters based on modern techniques. MATLAB 2011 software is chosen for the implementation of the algorithm which will be accessible through a GUI (Graphical User Interface) making the result of the investigation easily accessible.

2. MATHERIALS AND METHODS

In order to implement the desired algorithm, the authors first conducted a review of numerous articles related to the topic. It was noted that there was no single methodology applied when fitting clutter models to empirical data. However, some methods were definitely more frequently used than the rest. Therefore, the authors selected the most popular methods for creating the algorithm for estimating the sea clutter distribution and parameters. These methods are explained in the current section.

The overall structure of the implemented algorithm is shown in Figure 1. Basically, it consists of 7 steps, of which the first two are not applied when operating in real environments. Nevertheless, for testing the algorithm in a laboratory environment it was necessary (step 1) to enter the clutter parameters and with them (step 2) to generate random samples subordinated to Rayleigh, Log-Normal, Weibull and K models. Thus, the tests were performed knowing beforehand that the correct result was generated in steps 1 and 2. Note that in the real operation mode, the committed error cannot be known in advance. In fact, the goal of the algorithm is to estimate the model introduced in step 2 and the parameters used in step 1.

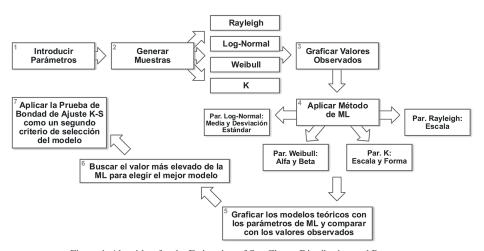


Figure 1. Algorithm for the Estimation of Sea Clutter Distribution and Parameters.

The first step that it's to be performed in real mode operation is the third-one from Figure 1, called "Plot Observed Values". It aims to accommodate the amplitude samples into histograms for further processing. The histogram is adopted as an intermediate step as it's done often in several related papers (12, 7, 15).

Then, in step 4, the ML (Maximum Likelihood) method is applied to estimate the distributions' parameters. The ML has been applied in numerous studies (27-30, 8) as an accurate but computationally expensive method. The authors chose the ML, while recognizing another viable alternative in the method of moments addressed by (31, 8, 15).

The fifth step is to plot the theoretical models found using the ML estimates and compare them with the histogram put-together from generated samples. If the comparison is made in a visual way, an intuitive idea may be obtained on which model is more suitable. However, the ML method itself offers a measure of the deviation of each distribution; which is exploited in the sixth step where the highest ML is chosen as an indicator of the preferential model.

Another alternative for choosing the best suitable model is to use an statistical tests. The Chi-Square (28, 32, 33) and the Kolmogorov-Smirnov (K-S) (34, 35, 21) tests are the most frequently used in the literature. Even though both options are acceptable, the authors chose to implement the K-S test in step 7 because they noticed, after performing several trials, that it was more accurate at identifying the proper distribution; while the Chi-Square test generally accepted as valid more than a single distribution in cases where resemblance levels were high.

The previous explanation covers the implemented algorithm superficially. In the following sub-sections the fundamental aspects will be detailed. First, the appropriate mathematical expressions for distributions used in clutter modeling will be presented; later the method of ML will be discussed; and the section will end with a small description of the K-S statistic test.

2.1. Clutter distributions

The Rayleigh, Log-Normal, Weibull and K models were used for clutter modeling. Their PDFs (Probability Density Functions), CDFs (Cumulative Density Function) and generating functions are introduced below.

For the Rayleigh distributio (31):

$$pdf(x;a) = \frac{x}{a^2} * \exp(-\frac{x^2}{2a^2})$$
 (1)

$$cdf(x; a) = 1 - \exp\left(-\frac{x^2}{2a^2}\right)$$
 (2)

$$F^{-1}(u) = \sqrt{(-2a^2 * \ln(1-u))}$$
 , $0 < u < 1$ (3)

Where (a) it is the scale parameter of the distribution and (u) is a uniformly distributed variable.

For the Log-normal distribution (31, 36):

$$pdf(x;\sigma,\mu) = \frac{1}{x\sigma\sqrt{2\pi}} * \exp\left(-\frac{(In(x) - \mu)^2}{2\sigma^2}\right); x \ge 0 \tag{4}$$

$$cdf(x;\sigma,\mu) = \frac{1}{2} + \frac{1}{2} * \operatorname{erf}\left(\frac{\ln(x) - \mu}{\sqrt{2\sigma^2}}\right)$$
 (5)

$$F^{-1}(u) = ex \, p \left(\mu + \sqrt{2\sigma^2} * erf^{-1}(2u - 1) \right) \ , \ 0 < u < 1 \eqno(6)$$

Where (μ) and (σ) are respectively the scale and shape parameter of the model matching the first two moments of the associated normal distribution; and (erf) is the error function.

For the Weibull Distribution (31, 36):

$$pdf(x; h, v) = \frac{2h}{\Gamma(v)} * \left(\frac{hx}{2}\right)^{v} * k_{v-1}(hx)$$
 (7)

$$cdf(x; h, v) = 1 - \frac{2}{\Gamma(v)} * \left(\frac{hx}{2}\right)^{v} * k_{v}(hx) ; 0 \le x < \infty$$
 (8)

$$F^{-1}(u) = \alpha [-\ln(1-u)]^{\frac{1}{\beta}}, \ 0 < u < 1$$
 (9)

Where (α) it's the scale parameter and (β) the shape parameter.

For the K Distribution (37):

$$pdf(x; h, v) = \frac{2h}{\Gamma(v)} * \left(\frac{hx}{2}\right)^{v} * k_{v-1}(hx)$$
 (10)

$$cdf(x; h, v) = 1 - \frac{2}{\Gamma(v)} * \left(\frac{hx}{2}\right)^{v} * k_{v}(hx) ; 0 \le x < \infty$$
 (11)

To obtain clutter samples from the K distribution, two independent random samples must be combined: one representing the slow variation and the other-one the clutter fast fluctuating component:

$$z(t) = x(t) * y(t) \tag{12}$$

A detailed description of the process of generating K distributed samples is offered in (37), together with the corresponding MATLAB code. The K models depends on the scale parameter (h), the shape parameter (v), the gamma function (Γ) and the modified Bessel function of the second kind of order v (k_{v-1})..

2.2. Maximum likelihood method

According to the algorithm shown in Figure 1, the Maximum Likelihood (ML) method is used in step 4 for searching the parameters of each model that maximize the similarity with the generated data. Subsequently, in step 6, the theoretical distribution with the highest value is selected as the preferential model.

In general, the ML method is recognized as a standard approach to the determination of a distribution parameters. It is said that the estimates obtained are optimal in the sense that they represent the most likely parameters for the given data and a theoretical PDF, if no prior knowledge is available (38, 39).

If the samples $x_1, x_2, ..., x_n$ are available for obtaining m parameters $\theta_1, \theta_2, ..., \theta_m$ from a given distribution, the joint PDF will be the multiplication of the independent PDFs:

$$f(x_1, x_2, \dots, x_n; \theta_1, \theta_2, \dots, \theta_m) = \prod_{i=1}^n f(x_i; \theta_1, \theta_2, \dots, \theta_m)$$
(13)

The estimates $\theta_1, \theta_2, ..., \theta_m$ that maximize the previous joint PDF are known as ML estimates. As a general rule, the estimates $\theta_1, \theta_2, ..., \theta_m$ can be found using the Lagrange method, which exploits the fact that the logarithmic function is monotonous and turns the product into a sum. Thus, ML estimates can be obtained by:

$$\begin{cases} \frac{\partial}{\partial \theta_1} \sum_{i=1}^n \log f(x_i; \theta_1, \theta_2, \dots, \theta_m) = 0 \\ \frac{\partial}{\partial \theta_2} \sum_{i=1}^n \log f(x_i; \theta_1, \theta_2, \dots, \theta_m) = 0 \\ \dots \\ \frac{\partial}{\partial \theta_m} \sum_{i=1}^n \log f(x_i; \theta_1, \theta_2, \dots, \theta_m) = 0 \end{cases}$$
(14)

In (40), more specific ML expressions are given for the distributions considered in this study. In addition, some derived alternatives are discussed.

2.3. Kolmogorov-Smirnov Test

Given a theoretical distribution with its parameters estimated for a particular data set, the goodness of fit Kolmogorov-Smirnov (KS) test (41, 42) can be performed to

determine if the data actually came from the theoretical distribution (see step 7 of the algorithm presented in Figure 1). The test is based on finding the separation between the experimental and hypothetical CDFs. Following the notation used by (43), the test can be divided into three steps test:

- 1. Calculate $D_n = max |F_i^* F(x)|$. Where F_i^* are the accumulated relative frequencies for each of the n values of the sample set and F(x) are the expected theoretical cumulative probabilities.
- 2. Compare D_n found in the previous step with D_n (1- α) which is a limit provided in the Kolmogorov tables for a given confidence level of 1- α .
- 3. If the D_n found in step 1 is $< D_n$ (1- α) from tables, then the null hypothesis (H_0) , which states that samples belong to the theoretical distribution, is not rejected with a confidence level of 1- α . If the D_n found in step 1 is $> D_n$ (1- α) from tables, then H_0 is rejected with an error margin of α (α is generally equal to 5% or 1%).

3. RESULTS AND DISCUSSION

After presenting the methods used for assembling the algorithm exposed in Figure 1, this section is devoted to provide evidence of its implementation and proper functioning. Figure 2 displays the result of generating samples (step 2) from Rayleigh, Lognormal, Weibull and K distributions. As it can be seen, there is no remarkable difference between the data sets when these are observed in a time series. Note that the number of samples is 2000 in each case. The authors determined, by performing several trials, that the value was high enough to ensure an accurate identification of a particular distribution.

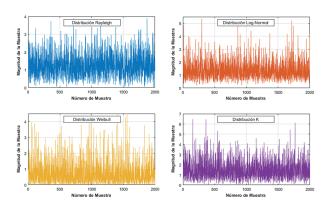


Figure 2. Time series of data sets with 2000 samples.

After applying the ML method (Step 4) and rearranging data sets into histograms, plots similar to figure 3 are obtained. Then, a visual comparison may provide a first approximation to the selection of the preferential distribution. Note in figure 3 that Weibull and K theoretical PDFs (blue) are very similar to data histograms (green) for the given example.

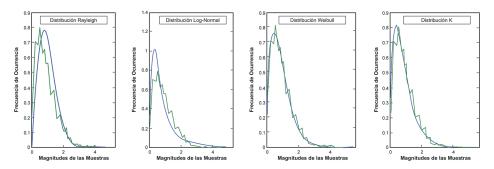


Figure 3. Visual comparisons between histograms from observed values (green) and hypothetical PDFs.

Usually, the best fitting model can't be selected by simple visual inspection. Therefore, the implemented algorithm includes two alternatives for making the final decision: the value of the ML estimation and the K-S statistical test. Tables I and II show possible results when applying these methods.

Table I. An example of obtained results when applying the Maximum Likelihood Method.

Distribution	Weibull	Rayleigh	Log-normal	K
Result	-1604.5466	-1838.763	-1817.1256	-1625.1169

In Table 1, the highest value indicates the preferential distribution, corresponding to Weibull in the proposed example. The result indicates that data is more resemble to the Weibull ML estimation and therefore it's more likely to belong to that model. Note that the K model also receives a high value which is very close to Weibull, meaning a K approximation will also be suitable.

Table II. An example obtained when applying the Kolmogorov-Smirnov test.

Distribution	Weibull	Rayleigh	Log-normal	K
Result	Not false	False	False	Not false
Probability	0.97096	5.551e-034	5.79e-011	0.037961

The decision recommended by the ML method is confirmed in Table II after applying the K-S statistical test. The test assigns a probability of membership to each considered model. The Rayleigh and Log-normal distributions are related to very close to zero probabilities so they are easily ruled out. The Weibull model is recommended by the K-S test as it assures the samples belong to the Weibull distribution with a 97% of certitude. Finally, the K model receives a probability of about 3.8%, so the hypothesis that the samples belong to the theoretical K PDF cannot be completely ruled out if 1% it is taken as the decision threshold. Nevertheless, the K-S test clearly indicates the supremacy of the Weibull alternative for the proposed example.

Figure 3 and Tables 1 and 2 actually belong to an essay where Weibull samples were generated. The implemented algorithm has a similar behavior for the other distributions. However, it was found that K and Weibull models often show a high level of correspondence; while the Log-normal distribution is usually easily distinguish from the rest. In addition, the Rayleigh samples tend be confuse with the Weibull-ones, which is understandable since the first model is a particular case of the second.

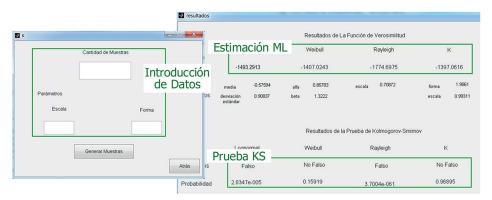


Figure 4. Graphical interface that allows the easy use of the implemented algorithm for the estimation of sea clutter parameters.

4. CONCLUSIONS AND FUTURE RESEARCH

An algorithm for the estimation of the distribution and parameters of sea clutter was implemented in MATLAB together with a GUI that facilitates pedagogical and research applications. The algorithm used seven steps for achieving the desired goal, among which the popular ML method and the Kolmogorov-Smirnov statistical test were incorporated. In addition, mechanism for generating Log-Normal, Rayleigh, Weibull and K distributions were included; so a tool for the full test of the ML method under variable sizes of the sample set was created. This tool can be used as comparison reference for novel radar detection solutions. The authors will focus next on the development of an improved version of this software, where the Method of Moments and more clutter distributions will be added.

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