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COSMOLOGICAL EXACT SOLUTIONS OF PETROV TYPE D. A MIXTURE OF TWO FLUIDS: DARK ENERGY AND RADIATION

SOLUCIONES COSMOLÓGICAS EXACTAS DEL TIPO PETROV D. UNA MEZCLA DE DOS FLUIDOS: ENERGÍA OSCURA Y RADIACIÓN

Rodrigo Alvarado Marín* Andrés A. Angulo Sibaja† Mauricio Vargas Obando‡

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*Universidad de Costa Rica, Centro de Investigaciones Espaciales (CINESPA) y Escuela de Física, San José, Costa Rica. E-Mail: rodrigo.alvarado@ucr.ac.cr

†Universidad de Costa Rica, Escuela de Física, San José, Costa Rica. E-Mail: andres.angulosibaja@ucr.ac.cr

‡Universidad de Costa Rica, Escuela de Física, San José, Costa Rica. E-Mail: mauricio.vargasobando@ucr.ac.cr

Abstract

In this paper, two exact solutions of the Einstein's equations are obtained for an anisotropic and homogeneous symmetry of Petrov Type D, the difference between both solutions lies in how relevant is the expansion that is presented initially, either on an axis or on a perpendicular plane. Both solutions represent a mixture of two fluids with minimum interaction: dark energy ($P = -\mu$) and radiation ($P = \mu/3$). The singularities and the influence that these fluids have on this metric are studied; the Hubble parameters, the deceleration parameter and the role that these fluids represent on them are determined and analyzed. Additionally, their temperature and the role that both play on this magnitude are determined.

Keywords: cosmology; exact solution; Einstein; temperature; Hubble; deceleration parameter; Kretschmann; singularity.

Resumen

En este trabajo se obtienen dos soluciones exactas de las ecuaciones de Einstein para una simetría anisotrópica y homogénea del Tipo Petrov D, la diferencia entre ambas soluciones radica en que tan relevante es la expansión que se presenta inicialmente, ya sea en un eje o en un plano perpendicular. Dichas soluciones representan una mezcla de dos fluidos con mínima interacción: energía oscura ($P = -\mu$) y radiación ($P = \mu/3$). Se estudia las singularidades y la influencia que estos fluidos tienen en esta métrica; así como el parámetro de Hubble, el parámetro de desaceleración y la influencia que estos fluidos tienen en ellos. Además, se determina su temperatura y el papel que ambos juegan en esta magnitud.

Palabras clave: cosmología; solución exacta; Einstein; temperatura; Hubble; parámetro de deceleración; Kretschmann; singularidad.

Mathematics Subject Classification: 83C15, 83C56, 83C75.

1 Introduction

The investigations related to the cosmic microwave background gathered by the COBE, WMAP and PLANK satellites, the accelerated expansion of the Universe, and all possible scenarios (states) in which the Universe could have passed, have generated a greater interest in Cosmology. These topics has been discussed in [1] in detail.

There are several studies of cosmological models in which the mixture of two or more fluids is considered, among them there is a model with three fluids with a FRW metric for a flat space-time [19]; for a mixture of radiation and matter it is studied the in-homogeneous cosmological solutions during the radiation era for a Lemaître-Tolman-Bondi metric in [18]; in [17] it has been studied two barotropic fluids without mutual interaction for dark energy models in a FRW universe, and in the same way [13] with this symmetry, it is analyzed a universe with dust and radiation fluids. Also, there have been efforts for quadratic state equations with three non-null terms representing vacuum energy, radiation and dark energy. Outside of the classic General Relativity theory in [6], it is developed a non-local modification of the theory with a new action S which allows to find cosmological solutions that represent interference properties between a radiation fluid and a dark energy one. In [4] cosmological solutions for an isobaric scalar field are presented for a homogeneous spacetime in a Petrov D symmetry. Using this symmetry, there has been obtained already a group of cosmological solutions for several types of fluids (see [1]) and it is the one used in this study to determine relevant cosmological functions for a particular mixture of fluids.

Such an analysis permits multiple scenarios to be studied within the same cosmological model, and it is used as a foundation in the investigation of possible characteristics by incorporating fluids, that partly represent the dark matter. Related to this line of investigation, here it is presented the solutions of a mixture of fluids with minimal interaction between them.

2 Symmetry, Einstein's equations and the solutions

The symmetry that will be used in this work is the anisotropic and homogeneous of Petrov Type D, which has the form [1]

$$ds^2 = F dt^2 - t^{2/3} K (dx^2 + dy^2) - \frac{t^{2/3}}{K^2} dz^2, \tag{1}$$

where F y K, are functions of t.

The components of the Einstein tensor ($G_\alpha^\beta = R_\alpha^\beta - \frac{1}{2}\delta_\alpha^\beta R$) different from zero, of (1), are

$$\begin{aligned}
G_0^0 &= \frac{4\,K^2 - 9\,t^2 \dot{K}^2}{12 t^2 K^2 F}, &(2)\\
G_1^1 &= -\frac{3Kt\dot{K}\left(2F - \dot{F}t\right) + 3Ft^2\left(2K\ddot{K} - 5\dot{K}^2\right) + 4K^2\left(\dot{F}t + F\right)}{12t^2K^2F^2}, &(3)\\
G_2^2 &= G_1^1 = -\frac{G_3^3}{2} + \frac{9\,Ft^2\dot{K}^2 - 4\,K^2\dot{F}\,t - 4\,K^2 F}{8t^2K^2F^2}, &(4)
\end{aligned}$$

where the dots over the functions represent time derivates.

The model of the perfect fluid used in cosmology represents a fluid without viscosities, isentropic ($P = P(\mu)$) and without shear stress, which could be written in the following way

$$T_{\alpha\beta} = (\mu_T + P_T)\, u_\alpha u_\beta - g_{\alpha\beta} P_T, \tag{5}$$

where $T_{\alpha\beta}$ is the energy momentum tensor of the mixture of fluids with minimal interaction, u_α the tetradimensional velocity, $g_{\alpha\beta}$ the metric tensor, μ_T and P_T the total energy density and the total pressure of the mixture of fluids, respectively.

The form of the equation of state for the mixture of fluids that will be used is

$$P_T = P_{eos} + P_{rad} \cdot \mu_T = \mu_{eos} + \mu_{rad}, \tag{6}$$

where μ_{eos} and μ_{rad} represent the energy densities of dark energy and radiation fluids, respectively, and P_{eos} and P_{rad} their respective pressures.
It will be considered a fluid with a tetra-dimensional velocity $u_\alpha = (u_0, 0, 0, 0)$; hence, the components of the energy momentum tensor (5), different from zero, are $T_0^0 = \mu_T = \mu_{eos} + \mu_{rad}$ and $T_1^1 = T_2^2 = T_3^3 = -P_T$ implying that from Einstein's equations $G_\alpha^\beta = \kappa T_\alpha^\beta$ must be satisfied that $G_1^1 = G_3^3$; thus, of (3) and (4) it is obtained

$$\dot{K}K\left(2F - \dot{F}t\right) - 2Ft\left(-K\ddot{K} + \dot{K}^2\right) = 0, \tag{7}$$

therefore

$$K = K_0 e^{C_1 \int \frac{F^{1/2}}{t} dt}, \tag{8}$$

without losing generalities, the constant K_0 in (8) will be considered equal to 1 and $C_1 = \pm 2/3$; for each possible value of C_1, a different model is obtained.

In [1], it was determined that the solution of fluids with lineal equations of state between P and μ of type $P = \lambda\mu$, for the anisotropic symmetry of Petrov D, gives a common result that

$$\mu_\lambda = \frac{C_\lambda}{t^{1+\lambda}} \quad \text{and} \quad P_\lambda = \frac{\lambda C_\lambda}{t^{1+\lambda}},$$

where C_λ is a constant of integration that represents the type of fluid depending on the value of λ. For the case under analysis, it will be defined as $\lambda = -1$ and $C_{-1} = \Lambda$ for dark energy and $\lambda = 1/3$ and $C_{1/3} = \mathcal{R}$ for radiation according to the equations of state usually analyzed in the literature. Once replaced these values, the following equations for the combined fluids are found

$$P_T = -\Lambda + \frac{\mathcal{R}}{3t^{4/3}}, \tag{9}$$

and the density μ_T

$$\mu_T = \Lambda + \frac{\mathcal{R}}{t^{4/3}}. \tag{10}$$

From Einstein's equations $G_\alpha^\beta = \kappa T_\alpha^\beta$, with (8), (6) and from the equality $T^{\mu\nu}{}_{;\mu} = 0$ it is obtained, for any C_1, that the solution of F, matching (10) with (2) is

$$F = \left(3\,\Lambda t^2 + 3\,t^{2/3}\mathcal{R} + 1\right)^{-1}, \tag{11}$$

where $\Lambda > 0$ and $\mathcal{R} > 0$; by cancelling one of these parameters, only one of the fluids remain. The numerical values for Λ and $\mathcal{R}$ (without units) in a unit of time are equivalent to the energy density of the respective fluid.

The function K in (8), once solved in its integral, takes the following form

$$K = \mathrm{e}^{\pm 8\,\frac{\sqrt{3}\Lambda}{C_2}\sqrt{\frac{A_1}{C_1}}\,\Pi\left(1/12\,\sqrt{2}\sqrt{\frac{-i12^{2/3}\sqrt{3}B_1}{F_1}},\frac{2\,i\sqrt{3}\sqrt{F_1}}{C_2},\sqrt{2}\sqrt{\frac{i\sqrt{3}F_1}{C_1}}\right)\pm 2/3\,\sigma_0}, \tag{12}$$

where

$$\begin{aligned}
A_1 &= \left(\sqrt{3}\sqrt{\frac{4\,\mathcal{R}^3 + 3\,\Lambda}{\Lambda}} - 3\right)\Lambda^2, && (13)\\
B_1 &= \sqrt[3]{12}{A_1}^{2/3} + i\sqrt[3]{12}\sqrt{3}{A_1}^{2/3} + 12\,\Lambda t^{2/3}\,\sqrt[3]{A_1} - \Lambda 12^{2/3}\mathcal{R} \\
&\quad + i\Lambda\sqrt{3}12^{2/3}\mathcal{R}, && (14)\\
C_1 &= i\sqrt[3]{12}\Lambda\sqrt{3}\mathcal{R} + i\sqrt{3}{A_1}^{2/3} - 3\,\sqrt[3]{12}\mathcal{R}\Lambda + 3\,{A_1}^{2/3}, && (15)\\
C_2 &= i\sqrt{3}{A_1}^{2/3} + i\sqrt[3]{12}\Lambda\sqrt{3}\mathcal{R} + {A_1}^{2/3} - \sqrt[3]{12}\mathcal{R}\Lambda, && (16)\\
F_1 &= \sqrt[3]{12}\mathcal{R}\Lambda + {A_1}^{2/3}, && (17)\\
\sigma_0 &= 8\,\frac{\sqrt{3}\Lambda}{C_2}\sqrt{\frac{A_1}{C_1}}\Pi\,\times \\
&\quad \times\left(\frac{\sqrt{2}}{12}\sqrt{\frac{-i12^{2/3}\sqrt{3}B_0}{F_1}}, \frac{2\,i\sqrt{3}\sqrt{F_1}}{C_2}, \sqrt{2}\sqrt{\frac{i\sqrt{3}F_1}{C_1}}\right), && (18)\\
B_0 &= \sqrt[3]{12}{A_1}^{2/3} + i\sqrt[3]{12}\sqrt{3}{A_1}^{2/3} + 12\,\Lambda\,\sqrt[3]{A_1} - \Lambda 12^{2/3}\mathcal{R} + \\
&\quad i\Lambda\sqrt{3}12^{2/3}\mathcal{R}. && (19)
\end{aligned}$$

The function $\Pi(\nu, n, m)$ is an incomplete elliptic integral of the third kind, where ν is the sine of the amplitude, n is the characteristic, and m the parameter. Even though the above solution seems to be complex, it is possible to eliminate the complex term through the constant of integration σ_0.

2.1 Analysis of the function $K(t)$ when $\mathcal{R} = 0$

From (8), the function $K(t)$ can be integrated considering only dark energy ($\mathcal{R} = 0$) using the function of $F(t)$ from (11). The result is

$$K_{\pm} = \mathrm{e}^{\pm\frac{2}{3}\operatorname{artanh}\left(\frac{1}{\sqrt{3\,\Lambda\,t^2+1}}\right)+c_{\pm 1}}, \tag{20}$$

where $c_{\pm 1}$ is a constant of integration that depends of the sign in K; simplifying and making the following change in the time coordinate $t = \sinh\left(n\sqrt{3\Lambda}\right)/\sqrt{3\Lambda}$, the following expression for $K(t)$ is found

$$K_{\pm} = D_{\pm 1}\left(\frac{\cosh\left(n\sqrt{3\Lambda}\right) - 1}{\cosh\left(n\sqrt{3\Lambda}\right) + 1}\right)^{\pm 1/3}. \tag{21}$$

In this case, $D_{\pm}$ is an integration constant; that based on [1] is defined as $(\sqrt{3\Lambda}/4)^{\mp 1/3}$. By using (20), it is possible to approximate the value of K when $t \longrightarrow \infty$ and it is found that

$$K_{\pm} \approx D_{\pm}\mathrm{e}^{\mp 2/9\,\frac{\sqrt{3}}{\sqrt{\Lambda}t}}. \tag{22}$$

This expression is also obtained if K is approximate for nonzero Λ and $\mathcal{R}$, but with a different integration constant. This demonstrates that for long periods of time, the dark energy term dominates the Universe in both models in the same way. Furthermore, if the limit for this last expression is calculated towards infinity, then, a constant value for K is reached and a solution for a homogeneous and isotropic spacetime with dark energy of the FRW flat type is obtained. This is independent of the sign that is used in (8).

2.2 Analysis of the function $K(t)$ when $\Lambda = 0$

By using the same function in (8), but this time considering radiation only ($\Lambda = 0$) employing the equation of $F(t)$ from (11), the following solution for $K(t)$ is found

$$K_{\pm} = E_{\pm}\frac{\left(\sqrt{3\,t^{2/3}\mathcal{R}+1} \pm 1\right)^2}{3t^{2/3}\mathcal{R}}, \tag{23}$$

where $E_{\pm}$ is the constant of integration that depends on the sign selected in K. When $t \longrightarrow 0$, it is possible to approximate $K(t)$ to the following expression

$$K_{\pm} \approx E_{\pm}\left(\frac{3\mathcal{R}}{4}\right)^{\pm 1} t^{\pm 2/3}, \tag{24}$$

If this is compared to [1](equation 61), then, the constant is defined as $E_{\pm} = (3\mathcal{R}/4)^{\mp 1}$. Considering the positive values (+) of the function (24), it is found that a part of the metric space extends over the x, y plane and when the negative value is taken (-), the part of space of the metric lengthens along z and shrinks over the x, y plane. Additionally, this expression for K is the same for the mixed model if time tends to zero except for the constant E which demonstrates how the model with both fluids is reduced to the radiation one if it is close to $t = 0$.

3 Kretschmann invariant and singularities

To study the possible singularities of a given spacetime, the Kretschmann invariant is used. The importance of this invariant has been discussed in [1]. For the solutions found, the invariant has the form

$$\textit{Krets} = R_{\alpha\beta\gamma\tau}R^{\alpha\beta\gamma\tau} = \tag{25}$$

$$\frac{72\,t^2\mathcal{R}^2 \pm 32\,\sqrt{3\,t^2\Lambda + 3\,t^{2/3}\mathcal{R} + 1}t^{2/3} + 72\,t^{14/3}\Lambda^2 + 32\,t^{2/3} + 48\,t^{8/3}\Lambda + 48\,t^{4/3}\mathcal{R}}{27\,t^{14/3}},$$

The positive sign is taken when $C_1 = 2/3$ and the negative one if $C_1 = -2/3$. From the Kretschmann invariant (25), it is known that a singularity exists in $t = 0$ for any value of C_1. For the value of $C_1 = 2/3$, the singularity is equivalent to the Kasner E_{D_1} (with an order depth of t^{-4}) and if $C_1 = -2/3$, it is a singularity with a depth of $t^{-8/3}$ which is present due to the radiation fluid.

4 Hubble parameter and deceleration

The Hubble parameter H is defined for the FRW flat model solution as $H = \dot{\mathcal{A}}/\mathcal{A}$ where $\mathcal{A}$ is a scalar factor. Considering the Einstein's equations

$$R_{\mu\nu} - 1/2 g_{\mu\nu} R = kT_{\mu\nu}, \tag{26}$$

and taking into account the energy momentum tensor defined in (5) together with the same lineal state equation $P = \lambda\mu$, it is found that $\mathcal{A}(t) \approx t^{2/3(\lambda+1)^{-1}}$; then, the Hubble parameter is defined as

$$H = \frac{2}{3(\lambda+1)t}. \tag{27}$$

For the Bianchi-I symmetry,

$$ds^2 = dt'^2 - a^2 dx^2 - b^2 dy^2 - c^2 dz^2. \tag{28}$$

The Hubble parameter is defined as follows [2],

$$H = \frac{((abc)^{1/3})'}{abc}, \tag{29}$$

where $(\prime)$ represents derivate with respect to t'; a, b y c are scalar factors which depend of t', and the coordinates x,y and z respectively. Such parameter can be defined in general terms for a metric of the type

$$ds^2 = edt^2 - A^2 dx^2 - B^2 dy^2 - C^2 dz^2, \tag{30}$$

in the following way

$$H = \frac{((ABC)^{1/6})^{\cdot}}{e(ABC)^{1/6}}, \tag{31}$$

where e, A, B and C are functions of t and the dot over the function represents the derivative with respect to time. Based on this, the following solution for the Hubble parameter related to the metric under study is found

$$H = \frac{\sqrt{3\,\Lambda t^2 + 3\,t^{2/3}\mathcal{R} + 1}}{3t}, \tag{32}$$

where the components of the metric tensor have been considered A, B and C of (30) for the metric (1).

In regards to the deceleration, it is defined for the FRW solution as

$$q = -\ddot{\mathcal{A}}\mathcal{A}/\dot{\mathcal{A}}^2. \tag{33}$$

For the FRWL flat model, it is found that $\mathcal{A}(t) \approx t^{2/3(\lambda+1)^{-1}}$; therefore,

$$q = 1/2 + 3/2\lambda. \tag{34}$$

For the Bianchi-I symmetry, it is defined in the same way, but considering that $\mathcal{A} \to (abc)^{1/3}$, then, it can be written as

$$q = -1 - \frac{\dot{H}}{H^2}. \tag{35}$$

Using the respective values for the metric under study, it is obtained that

$$q = -\frac{3\,\Lambda t^2 - 3\,t^{2/3}\mathcal{R} - 2}{3\,\Lambda t^2 + 3\,t^{2/3}\mathcal{R} + 1}. \tag{36}$$

4.1 Results analysis for the Hubble parameter

The Hubble parameter, in the case of the combination of both fluids ($\Lambda \neq 0$ and $\mathcal{R} \neq 0$), tends to infinity when $t \to 0$ (it is independent of the value of C_1); it has the same behaviour even though one of the fluids is eliminated. When $t \to \infty$, $H \to H_0$, this tendency is present with both fluids or with dark energy only ($\mathcal{R} = 0$). For these two cases, $H_0 = \sqrt{3\Lambda}/3$ which demonstrates how the dark energy term dominates the late Universe's expansion. If only radiation is present (Λ=0), then, it is found that $H \to 0$.

4.2 Results analysis for the deceleration parameter

The deceleration parameter q tends to $q \to -1$ when $t \to \infty$ in combination of both fluids or with dark energy only ($\mathcal{R} = 0$), and it is independent of the value of C_1. This indicates that the universe accelerates after an era of deceleration in accordance with recent observations of distant objects [16, 10, 8]. If only radiation is present, ($\Lambda = 0$) the parameter $q \to 1$ indicating a permanent process of deceleration. If $t \to 0$, $q \to 2$; which imply a collapsed process for the mixture of fluids at the beginning. This same trend is kept if only dark energy is present ($\mathcal{R} = 0$) or only radiation ($\Lambda = 0$). (See Figure 1).

The time, in which the deceleration parameter becomes zero (described in (36)), can be defined as follows

$$t_1 = \frac{2\sqrt{6}}{9}\left(\sqrt{\frac{3\mathcal{R}}{\Lambda}}\cos\left(\frac{1}{3}\arccos\left(\frac{\sqrt{3\Lambda}}{\mathcal{R}^{3/2}}\right)\right)\right)^{3/2}, \tag{37}$$

and it represents the moment of time when the Universe expansion is not accelerating or decelerating. This point can be observed in the intersection function with the abscissa axis in the Figure 1 where the value of q changes from positive to negative representing the transition from deceleration to acceleration during the expansion.

In the case, only dark energy is present, it is considered $\mathcal{R} = 0$ and with the equation (36), the time for $q = 0$ is

$$t_2 = \frac{1}{3}\sqrt{\frac{6}{\Lambda}}, \tag{38}$$

and with only radiation with $\Lambda = 0$, there is no solution. Then, only decelerated expansion is present during all time of the Universe.

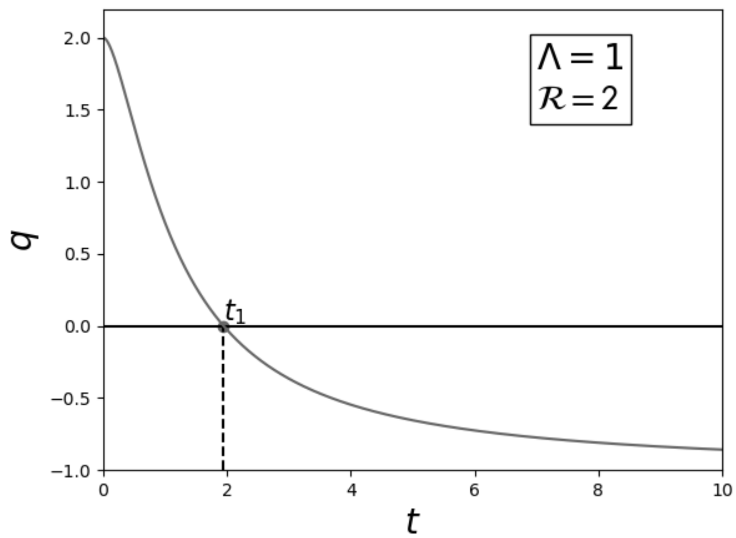

Figure 1: Curve of the deceleration parameter (q) with respect to time.

5 The temperature

In relation to thermodynamics in cosmological solutions with a symmetry of Petrov D, some possibilities have been discussed and analyzed in [3], where it has been determined with the symmetry of FLRW type that

$$\frac{dP_T}{\mu_T + P_T} = \frac{dT}{T}, \tag{39}$$

where T is the temperature of the mixture of fluids.
From (39), (10) and (9), it is obtained that

$$T = \frac{T_0}{t^{1/3}}, \tag{40}$$

where T_0 is a constant of integration. From (40), it is observable that for the mixture of both fluids when the time $t \longrightarrow 0$, the temperature $T \longrightarrow \infty$ and this result in the model is independent from the fluids constants. For bigger times as $t \longrightarrow \infty$, the temperature $T \longrightarrow 0$. When it is considered in (39) only radiation ($\Lambda = 0$), the same result is found as in (40), but if the same procedure is followed for dark energy ($\mathcal{R} = 0$), this solution is not found. Therefore, it can be concluded that the dependency of the temperature on time, in the mixture of these fluids, is determined by radiation.

6 Conclusions

In this study, solutions for a mixture of fluids were obtained; dark energy and radiation with minimal interaction and the possible combinations of both were analyzed. The model results were validated in the upper and lower time limits if there is one fluid, either dark energy or radiation, in accordance with previous studies [1]. It was determined that if $t \longrightarrow 0$ or $t \longrightarrow \infty$, the approximation of K for both fluids shows the same dependency from the time, with the only difference of a multiplying constant, to the model with only radiation (in $t \longrightarrow 0$) or dark energy (in $t \longrightarrow \infty$) respectively. Also, it was found that when the limit is calculated $t \longrightarrow \infty$, an isotropization of both fluids arises and if compared with the model with only dark energy, the only difference is the constant mentioned above.

It was found that for both models there is a singularity at $t = 0$ with an order depth of $t^{-8/3}$ if $C_1 = -2/3$ and a depth of t^{-4} (Kasner type E_{D_1}) if $C_1 = 2/3$.

For the Hubble parameter (H), it was estimated that if $t \longrightarrow 0$ for both fluids or only one of them, $H \longrightarrow \infty$ and if $t \longrightarrow \infty$, then, $H \longrightarrow H_0$, where $H_0 = \sqrt{3\Lambda}/3$.

The deceleration parameter $q \to -1$ if $t \to \infty$ in combination of both fluids or if only dark energy is present ($\mathcal{R} = 0$), this indicates an accelerated expansion of the universe in accordance with observations of Type la Supernovas [15, 14, 9]. If there is only radiation, ($\Lambda = 0$) the parameter $q \to 1$ indicates a deceleration process.
It was established that if $t \to 0$, $q \to 2$ which implies a deceleration process for the mixture of both fluids at the beginning. This same situation is shown if there is only dark energy ($\mathcal{R} = 0$) or only radiation ($\Lambda = 0$).

It was also defined the time instant (t_1) if the deceleration parameter is zero, representing the moment when there is no acceleration or deceleration and there is a transition among them. The time (t_1) was also calculated in the case of only dark energy, since when there is only radiation a decelerated expansion is experienced all the time. This transition was corroborated in the mid to late 1990s by the Supernova Cosmology Project and the High-z SN Search teams with Hubble's data of Type la Supernovas [16, 15, 14] and later by others investigations with data such as that of galaxy clusters [10].

It was demonstrated that the temperature, for the model with both fluids, shows a behaviour with high values at the beginning, but then $T \to 0$ if $t \to \infty$. The same result is obtained if there is only radiation ($\Lambda = 0$) which indicates the dominance of this parameter for the temperature. The existence of an isotropic radiation bath that permeates the entire Cosmos known as the Cosmic Microwave Background (CMB) discovered in 1964 [7] supports the theory of a universe with high temperatures at the beginning and then cooling off as it expands. The Cosmic Background Explorer satellite (COBE), launched in 1992 to look for minor changes in CMB temperature [12], and the Wilkinson Microwave Anisotropy Probe spacecraft (WMAP), launched in 2001, have both corroborated this. According to the WMAP survey, CMB is nearly identical in all directions [5].

Finally, it is worth noting that observations made by investigations such as [11] suggest evidence that challenges the well know Cosmological Principle, which shows the importance of pursuing more realistic models in accordance with the observable universe.

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